

Usage of HPC in simulations of charge transport in semiconducting materials

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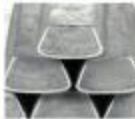
Motivation

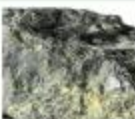
❖ Ohm's law

$$R = \rho L / S$$

❖ A lot of physics behind ρ



Проводници		
Материјали		
Бакар	Алуминијум	Гвожђе
		
Специфична отпорност [Ωm]		
$1,7 \cdot 10^{-8}$	$2,8 \cdot 10^{-8}$	$9,7 \cdot 10^{-8}$

Полупроводници		
Материјали		
Графит	Германијум	Силицијум
		
Специфична отпорност [Ωm]		
$3 \cdot 10^{-5}$ до $6 \cdot 10^{-4}$	0,001 до 0,5	0,1 до 60

Изолатори		
Материјали		
Дрво	Стакло	Пластика
		
Специфична отпорност [Ωm]		
10^8 до 10^{14}	10^{10} до 10^{14}	10^{13}

Materials by design

TOPICAL REVIEW • OPEN ACCESS

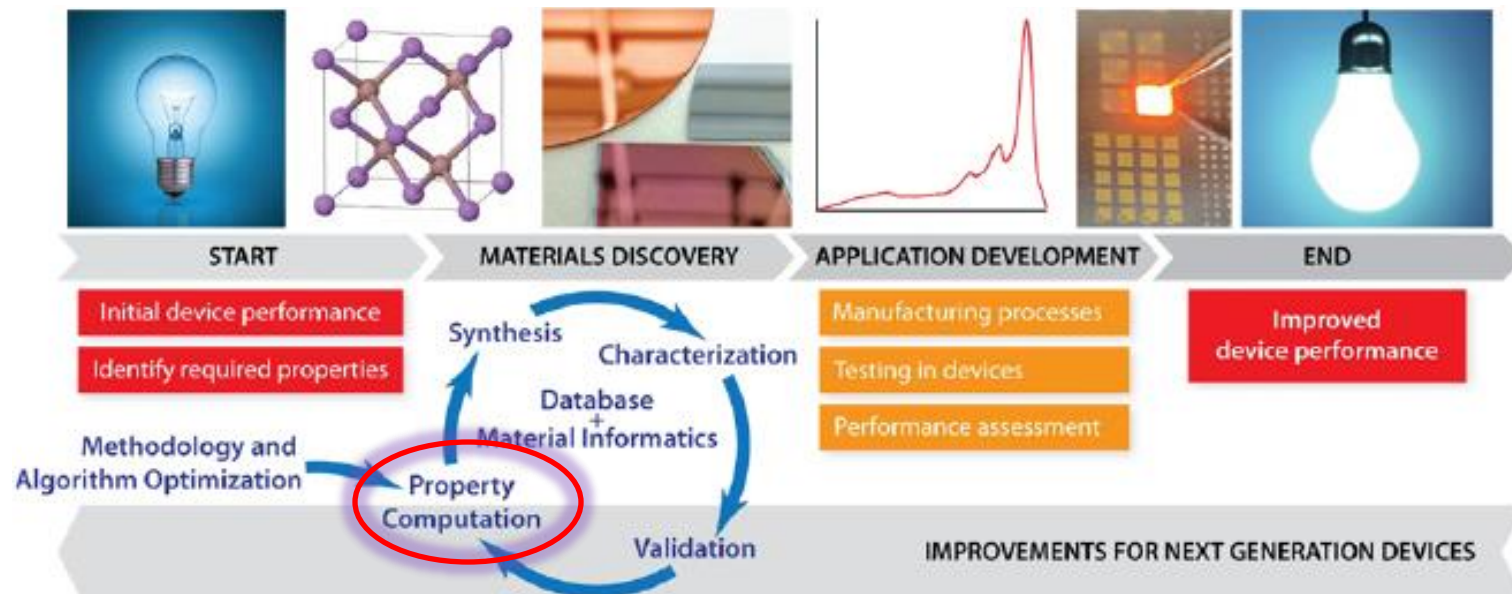
The 2019 materials by design roadmap

Kirstin Alberi¹ , Marco Buongiorno Nardelli², Andriy Zakutayev¹ , Lubos Mitas³,
Stefano Curtarolo^{4,5}, Anubhav Jain⁶, Marco Fornari⁷ , Nicola Marzari⁸, Ichiro Takeuchi⁹,
Martin L Green¹⁰ [+ Show full author list](#)

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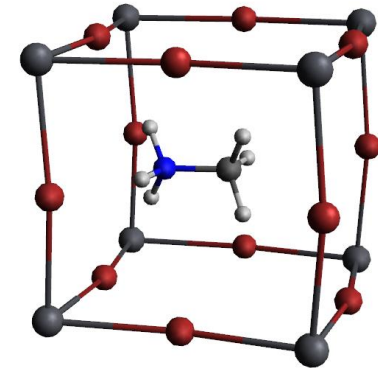
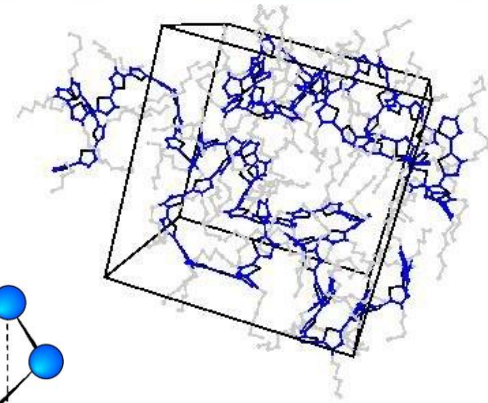
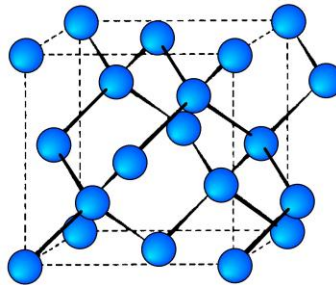
[Journal of Physics D: Applied Physics](#), Volume 52, Number 1

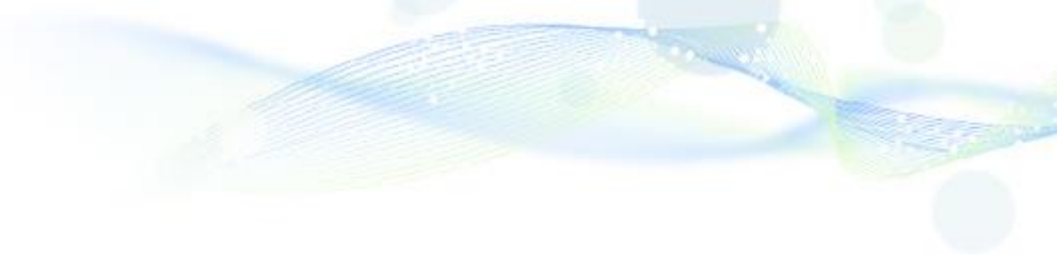
Citation Kirstin Alberi et al 2019 *J. Phys. D: Appl. Phys.* **52** 013001



Different charge transport mechanisms

- ❖ Hopping transport (e.g. disordered organic semiconductors relevant for organic cells)
- ❖ Band transport (crystalline semiconductors, e.g. silicon)
- ❖ Intermediate transport (emerging semiconductors, e.g. halide perovskites)
- ❖ Each of these mechanism calls for a different simulation / modeling approach

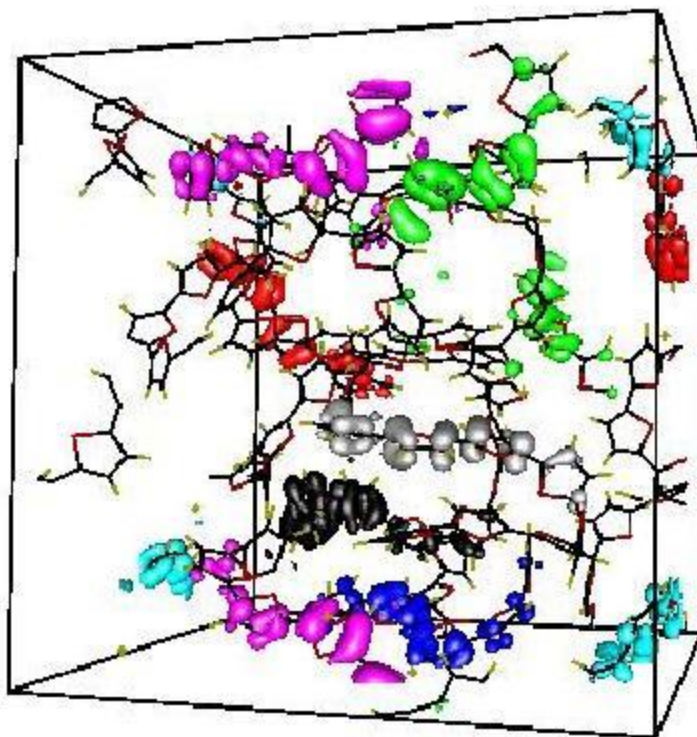




Hopping transport

Amorphous materials

- ❖ Localized electrons, move slowly.
- ❖ Large system calculations needed to extract transport properties.



N. Vukmirović and L. W. Wang, J. Phys. Chem. B 113, 409 (2009).

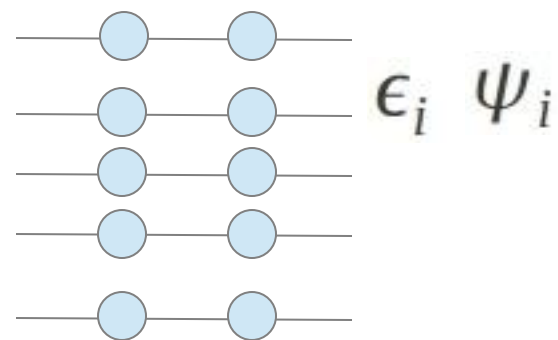
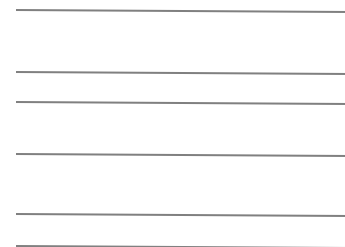
Density Functional Theory

❖ Limitation to ~ 100 -1000 atom systems.

$$\rho(\vec{r})$$

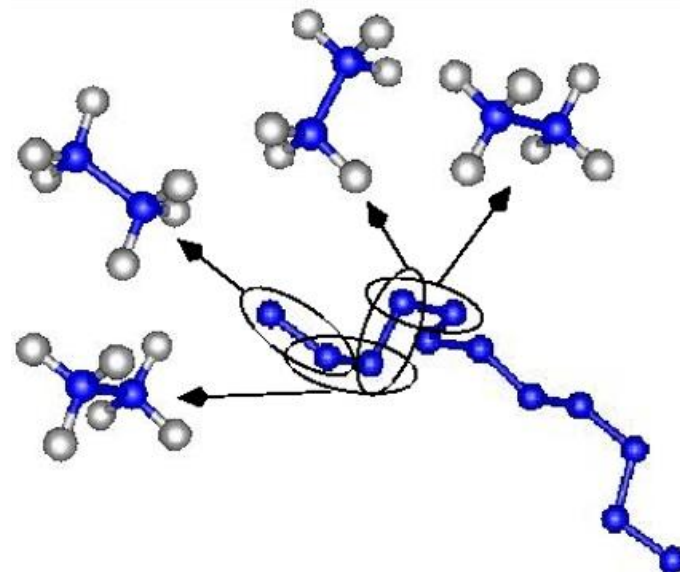
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U_{\text{eff}}(\rho)\right]\psi_i(\vec{r}) = \epsilon_i \psi_i(\vec{r})$$

$$\rho(\vec{r}) = -e \sum_{\text{occ}} |\psi_i(\vec{r})|^2$$

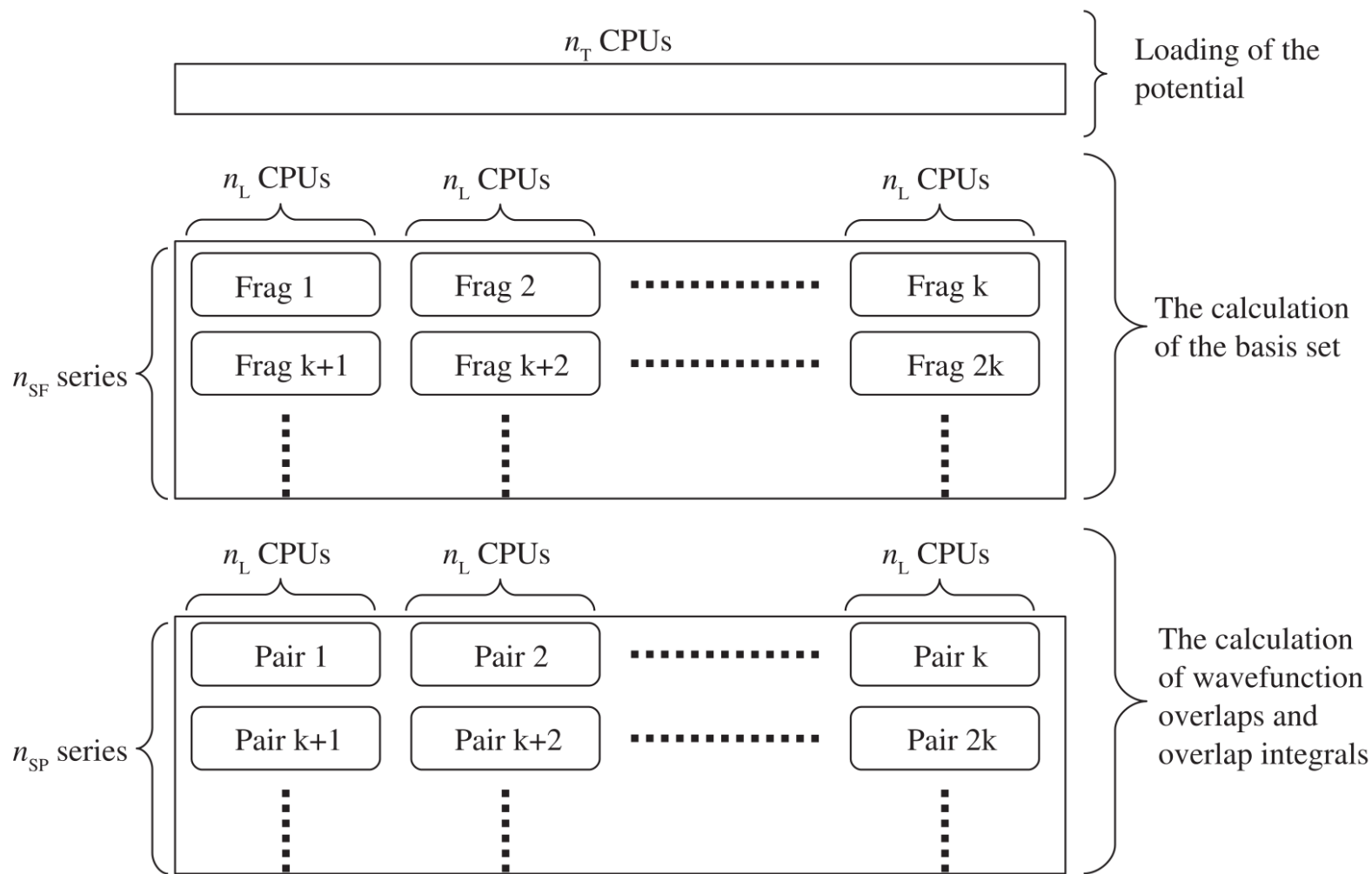


Overlapping fragments methods

- ❖ Division of the system into (mutually overlapping) fragments.
 - HOMOs (LUMOs) of the whole system linear combination of HOMOs (LUMOs) of fragments.
- ❖ Procedure:
 - Calculation of fragment orbitals ϕ_i
 - Calculation of $S_{ij} = \langle \phi_i | \phi_j \rangle$ and $H_{ij} = \langle \phi_i | H | \phi_j \rangle$
 - Solution of the generalized eigenvalue problem $\det(H_{ij} - ES_{ij}) = 0$

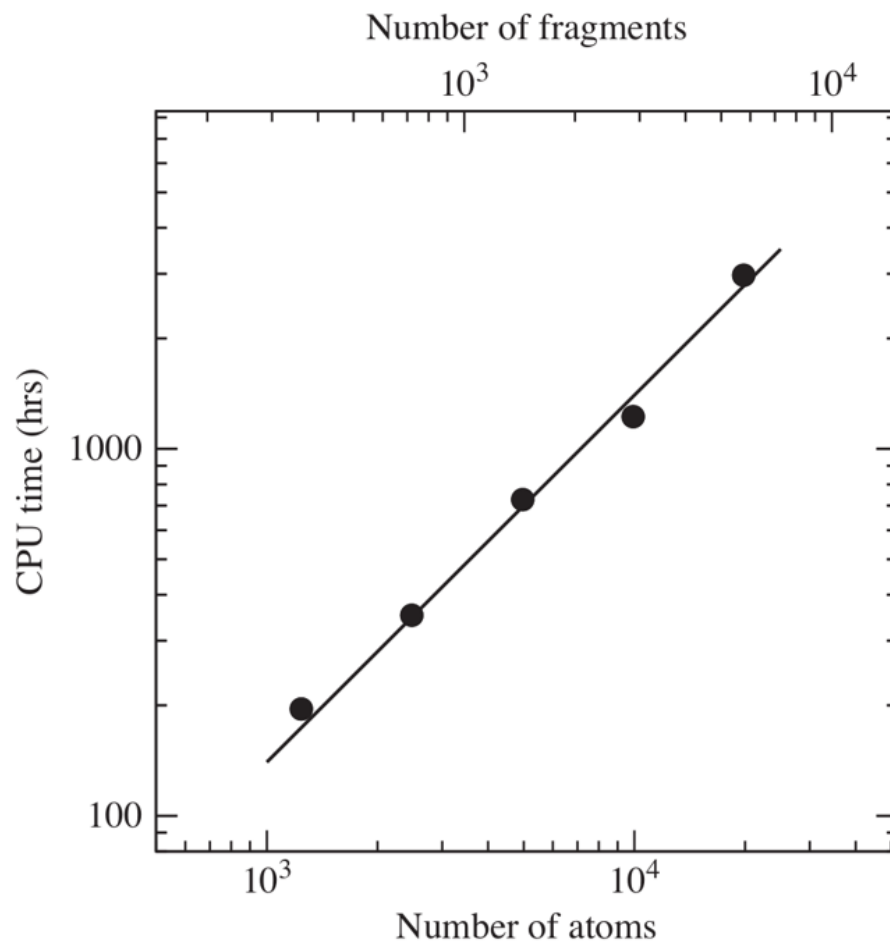


Overlapping fragments methods - implementation



N. Vukmirović and L.-W. Wang, J. Chem. Phys 134, 094119 (2011).

Overlapping fragments methods - scaling



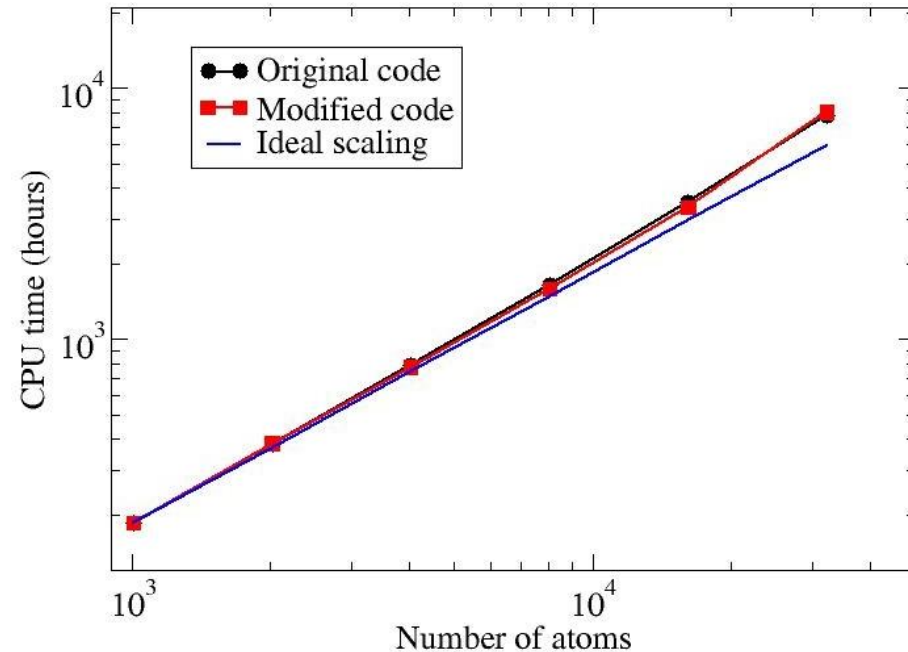
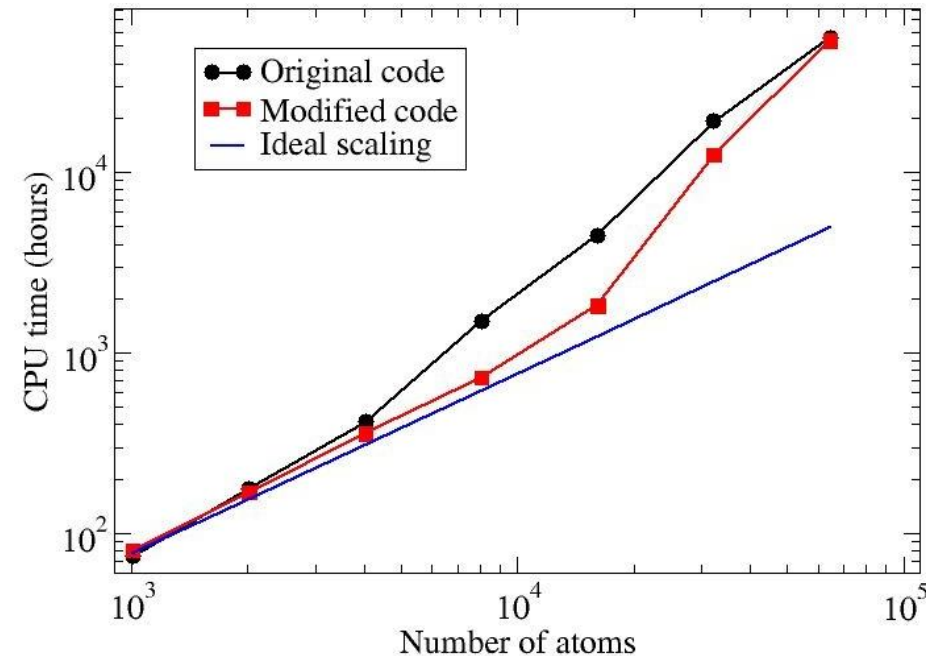
- Amorphous alkane system.
- Runs with typically 5000 CPU cores.
- One orbital per fragment.

N. Vukmirović and L.-W. Wang, J. Chem. Phys 134, 094119 (2011).

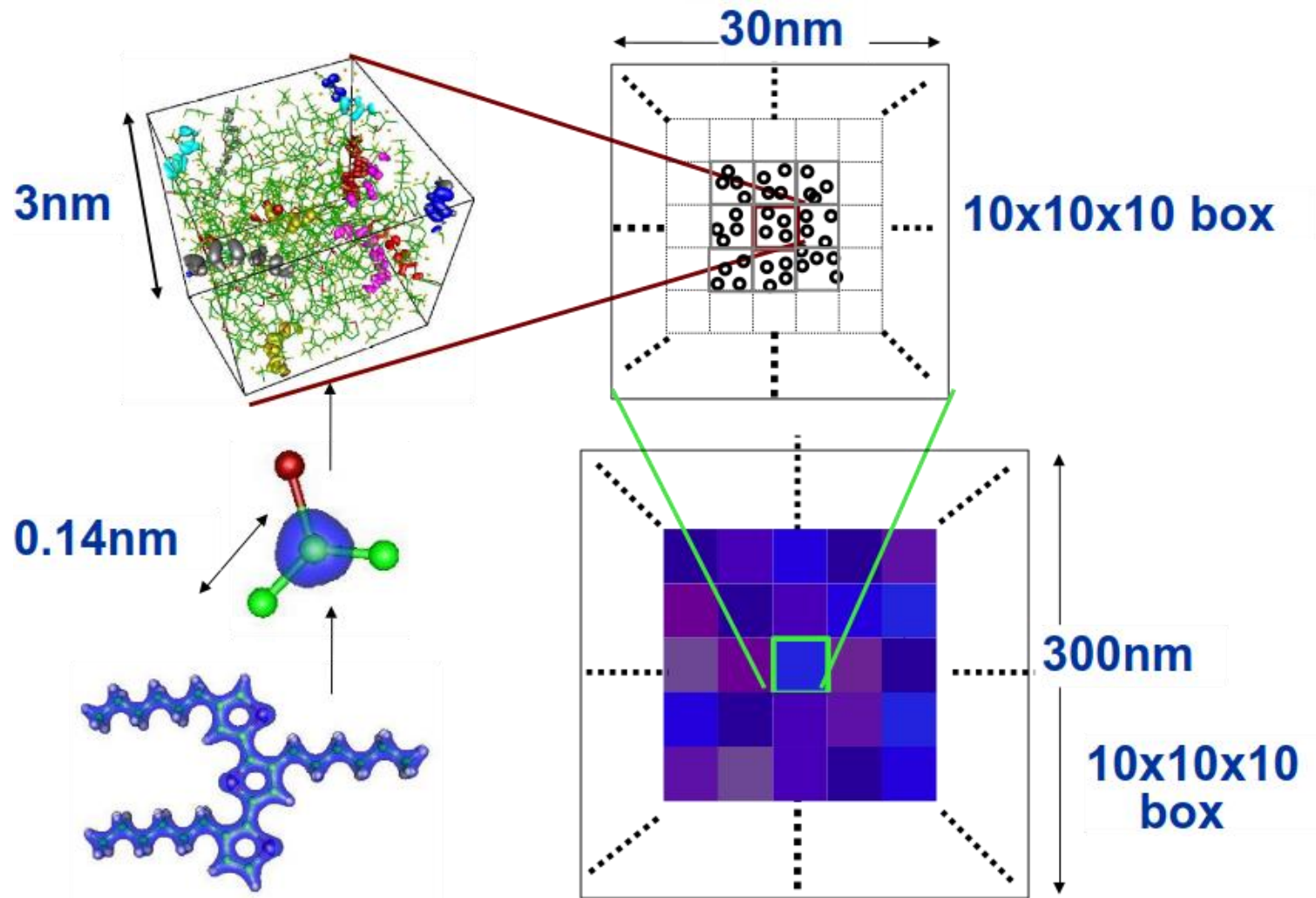
Overlapping fragments methods – role of communication

- Weak scaling test – Curie (Paris)
- Infiniband network.
- Tests using from 256 to 16384 cores.

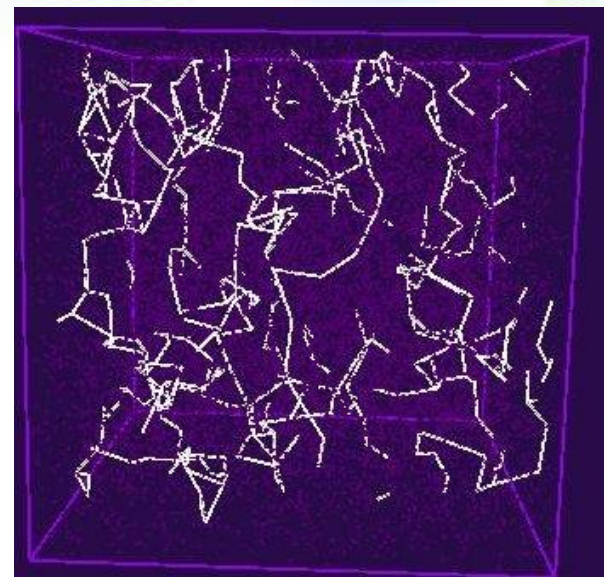
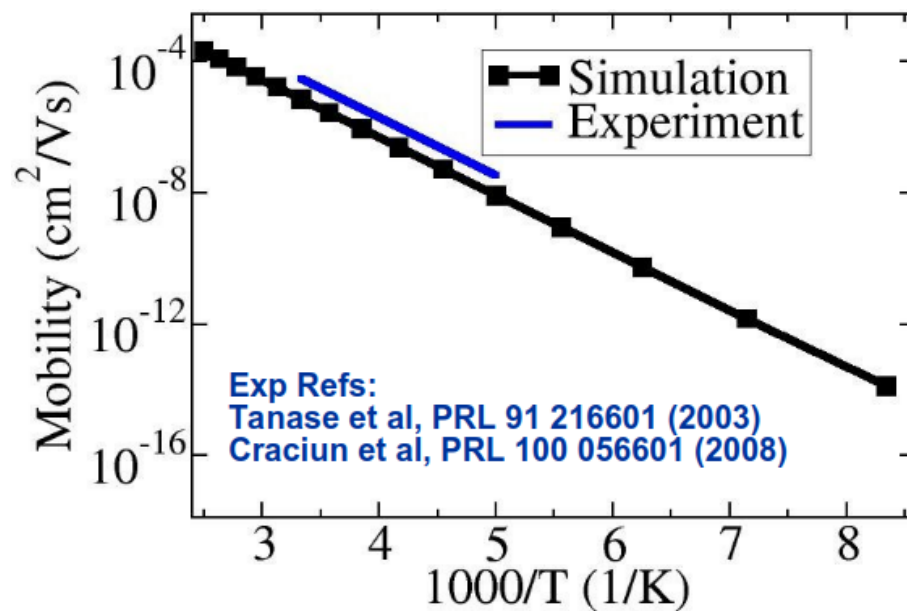
- Weak scaling test – HLRS (Stuttgart)
- Cray XE6 within Cray Gemini network.
- Tests using from 256 to 16384 cores.

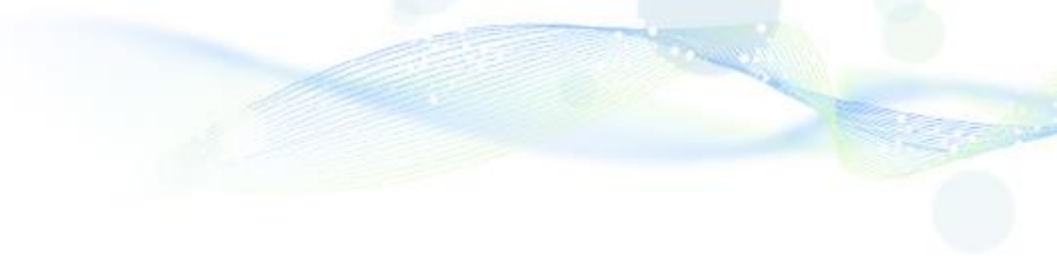


Multiscale method for carrier transport



Temperature dependence of mobility

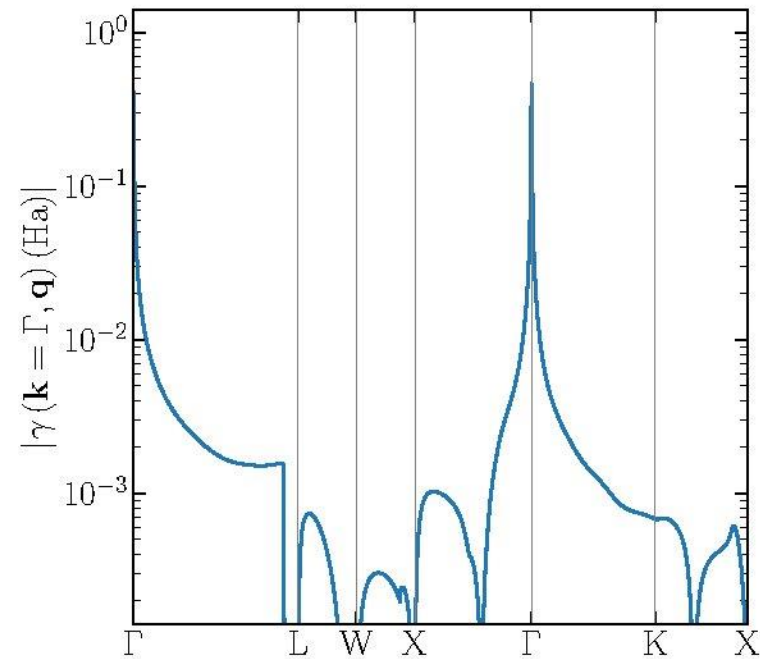
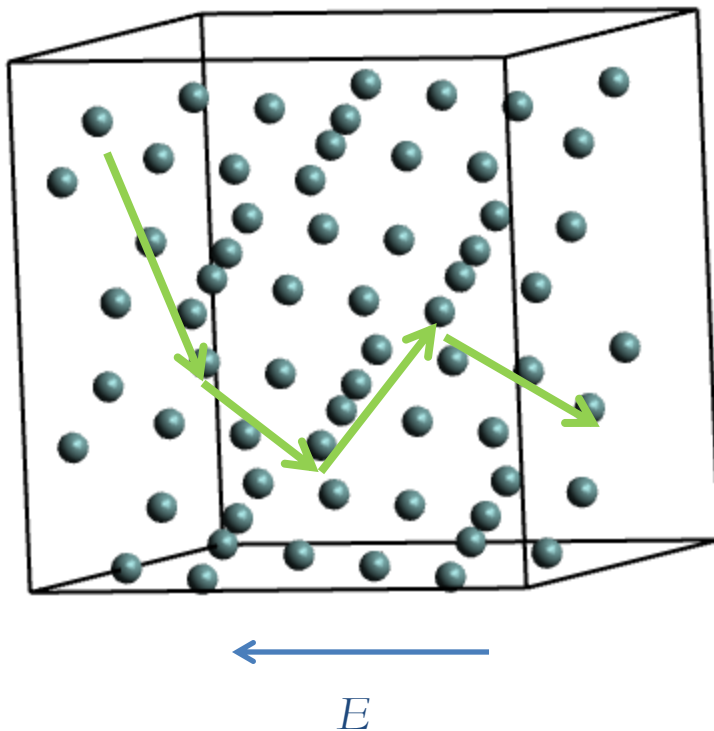




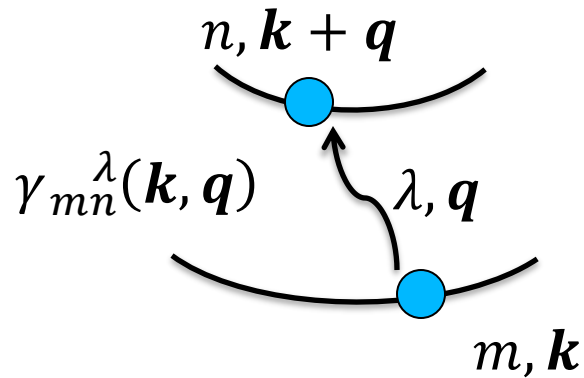
Band transport

Crystalline semiconductors

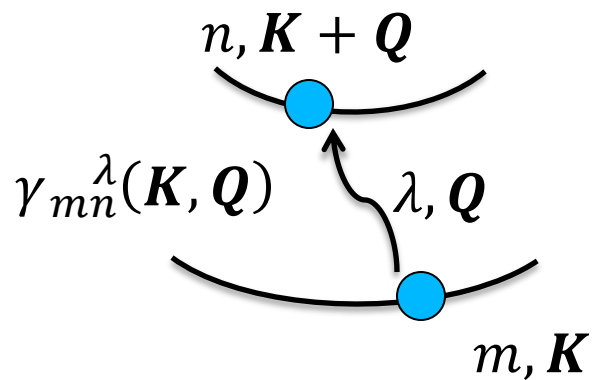
- ❖ Delocalized electron, moves easily.
- ❖ Lattice vibrations slow it down and limit its mobility.
- ❖ One needs to calculate electron-phonon coupling constants.
- ❖ These are strongly varying with q , so dense grids are needed.



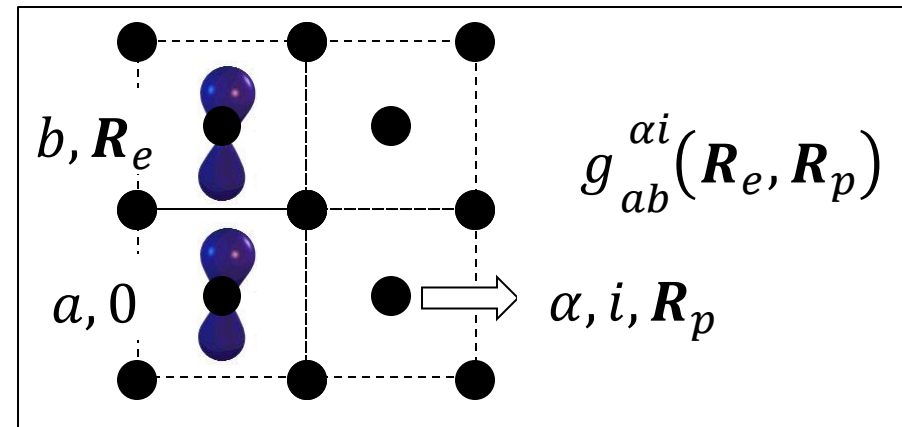
Electron-phonon coupling constants interpolation



$$\gamma_{mn}^{\lambda}(\mathbf{k}, \mathbf{q})$$



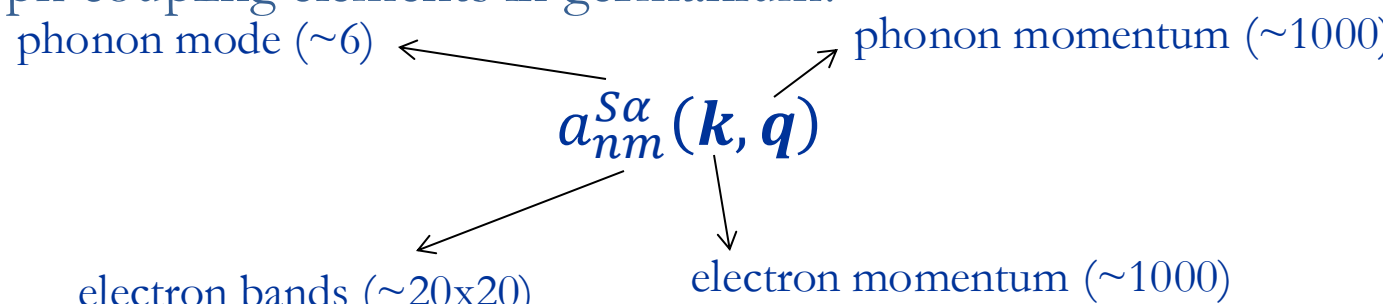
$$\gamma_{mn}^{\lambda}(\mathbf{K}, \mathbf{Q})$$



F. Giustino et al, PRB 76, 165108 (2007).

Need for large data structures

- ❖ Electronic states, phonon modes and e-ph coupling constants from ABINIT code. This is used as input to our parallel code.

- ❖ E-ph coupling elements in germanium:


phonon mode (~ 6) phonon momentum (~ 1000)

$a_{nm}^{S\alpha}(\mathbf{k}, \mathbf{q})$

electron bands ($\sim 20 \times 20$) electron momentum (~ 1000)

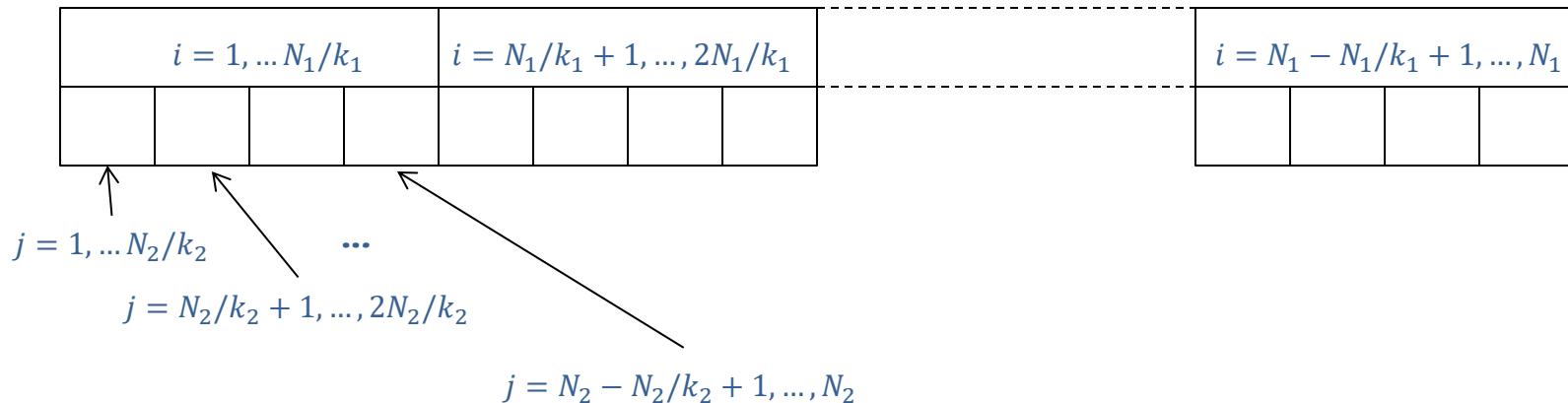
- ❖ One needs $\sim 40\text{GB}$ to store e-ph coupling elements on the course grid for germanium (2 atoms per unit cell)
- ❖ Distribution of these elements in memory of different nodes therefore necessary.

Data distribution

- ❖ Array a_i with N elements, machine with k cores.

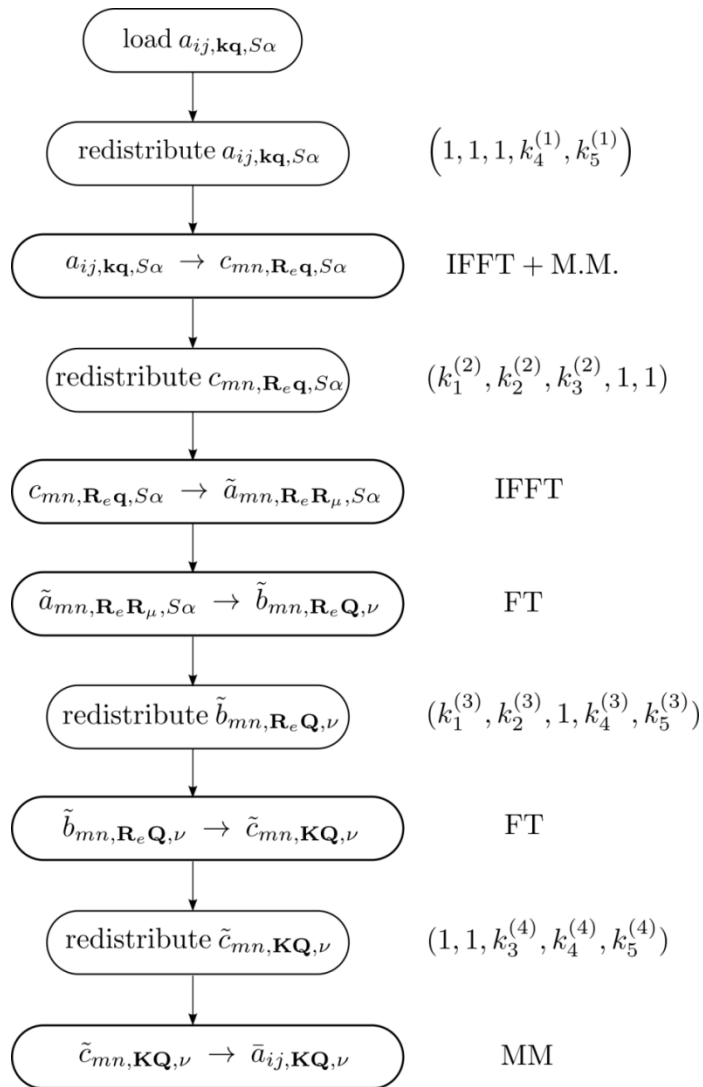


- ❖ Array a_{ij} with $N_1 N_2$ elements, machine with $k_1 k_2$ cores.



- ❖ Same approach to distribute the array a_{ijklm} with $N_1 N_2 N_3 N_4 N_5$ elements, on a machine with $k_1 k_2 k_3 k_4 k_5$ cores.

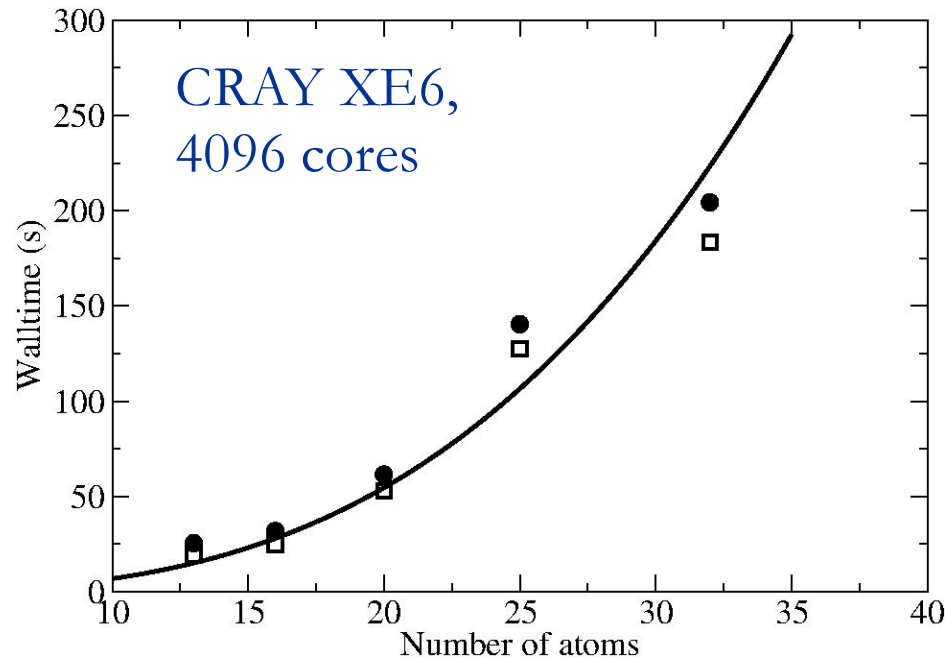
Parallelization



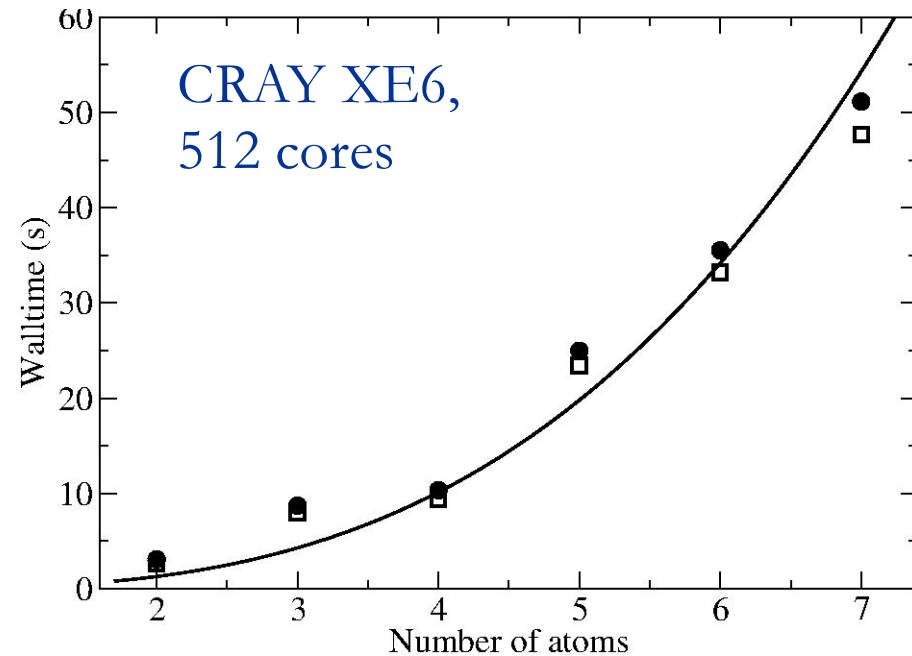
- ❖ The strategy is to have all array elements involved in the computation of FT or MM on the same core.
- ❖ To achieve this, we adopt redistribute – compute – redistribute approach.
- ❖ `mpi_alltoallv` used for redistribution.

Scaling with system size

- ❖ Parameters $n_b=2n_{at}$,
 $n_k=n_q=216$, $n_K=1$, $n_Q=200$



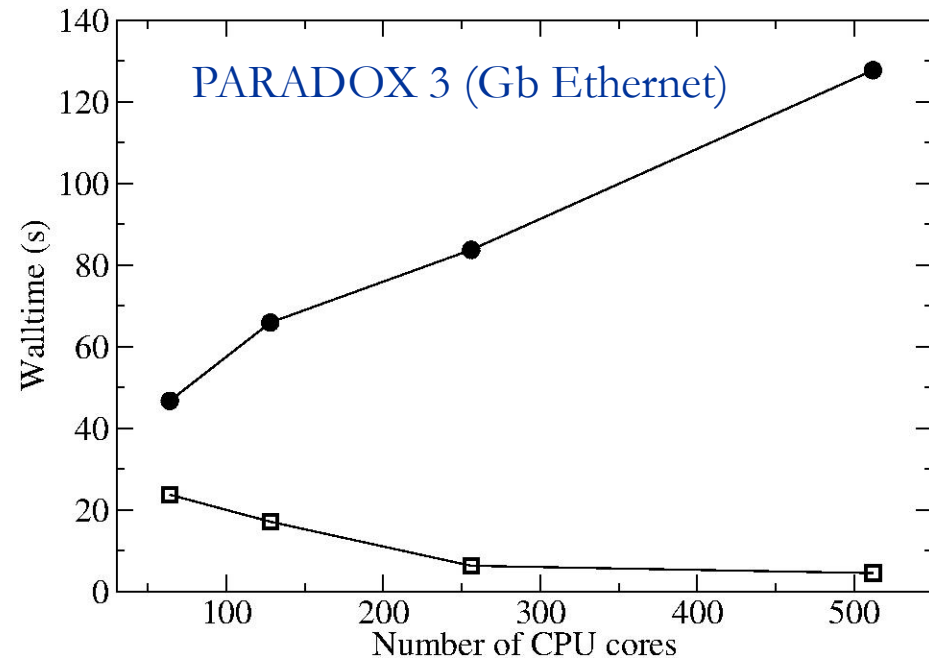
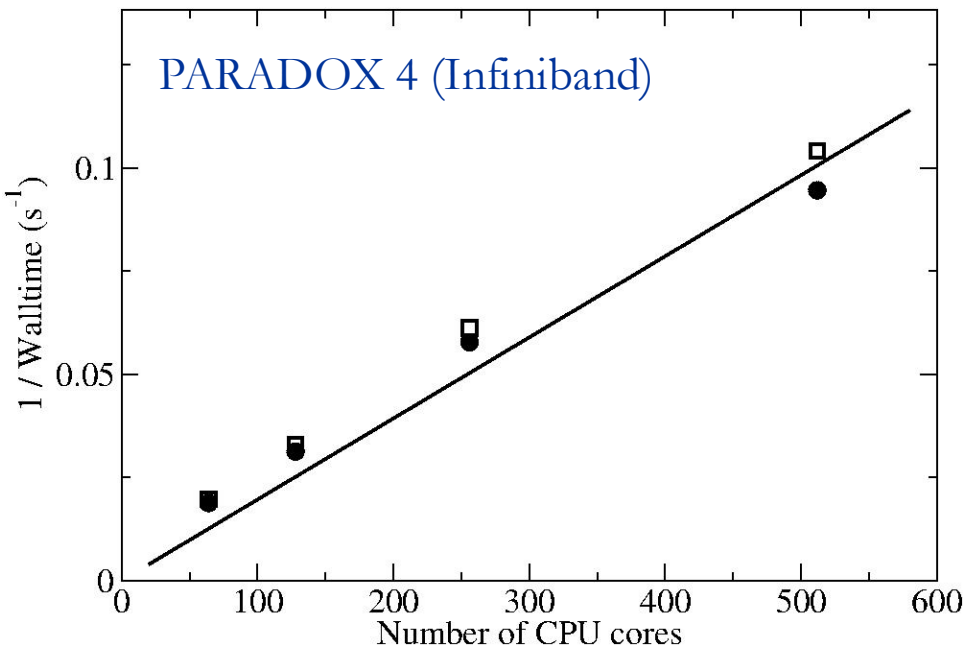
- ❖ Parameters $n_b=2n_{at}$,
 $n_k=n_q=216$, $n_K=1000$,
 $n_Q=200$



- ❖ $\sim n_{at}^3$ scaling with system size.
- ❖ communication negligible.

Scaling with number of CPU cores

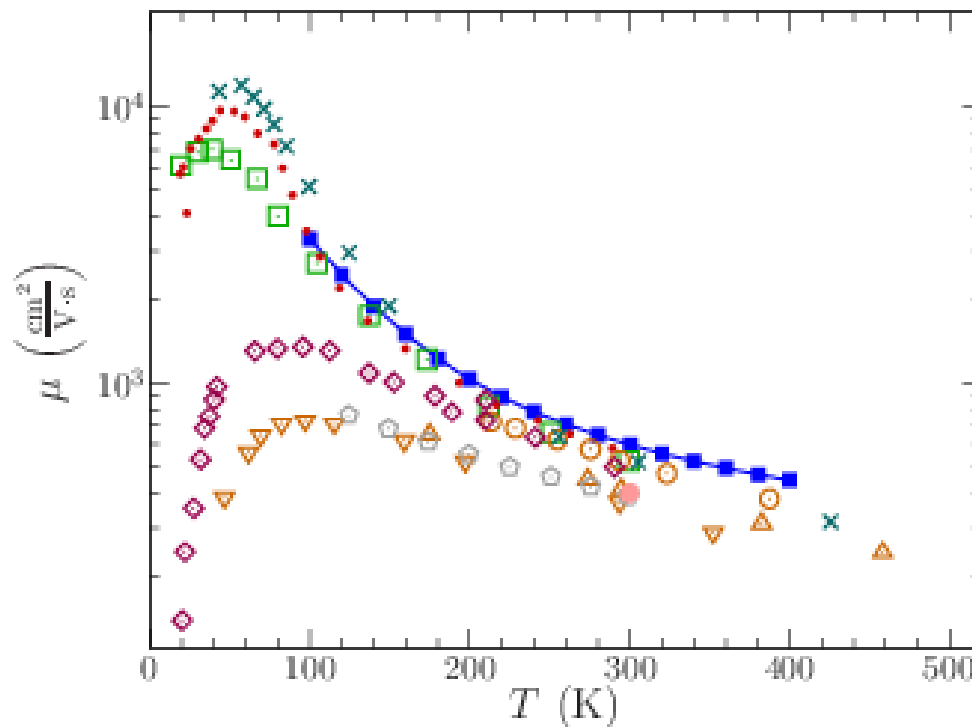
❖ Parameters $n_{at}=4$, $n_b=2n_{at}$, $n_k=n_q=216$, $n_K=1000$, $n_Q=200$



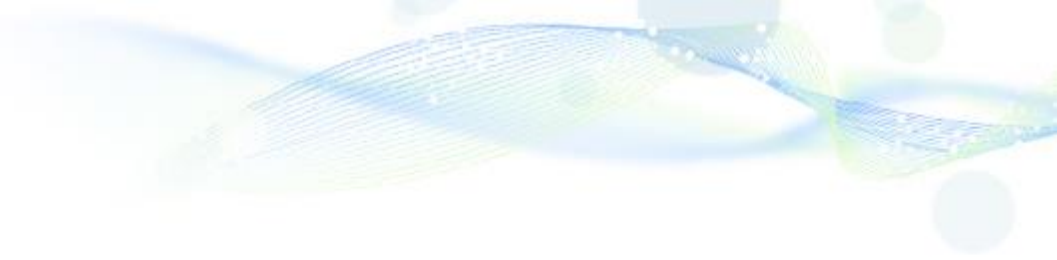
❖ linear scaling with system size on machine with infiniband network, poor scaling with Gb Ethernet.

Temperature dependence of mobility

- ❖ Mobility decreases with an increase in temperature.
- ❖ Example – ZnSe material, comparison with experiment.



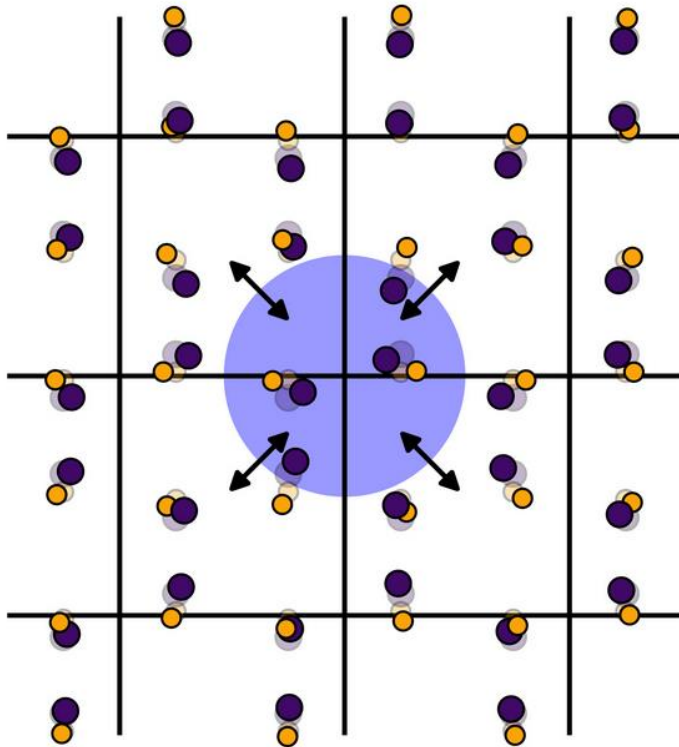
N. Vukmirović, Phys. Rev. B 104, 085203 (2021).



Intermediate regimes

Polarons in materials

❖ Polaron formation – mixed electron-phonon states.



- ❖ On going project:
Polaron Mobility in Model
Systems and Real Materials
(PolMoReMa – 5468)
(2024-2026).
- ❖ <http://polmorema.ipb.ac.rs>

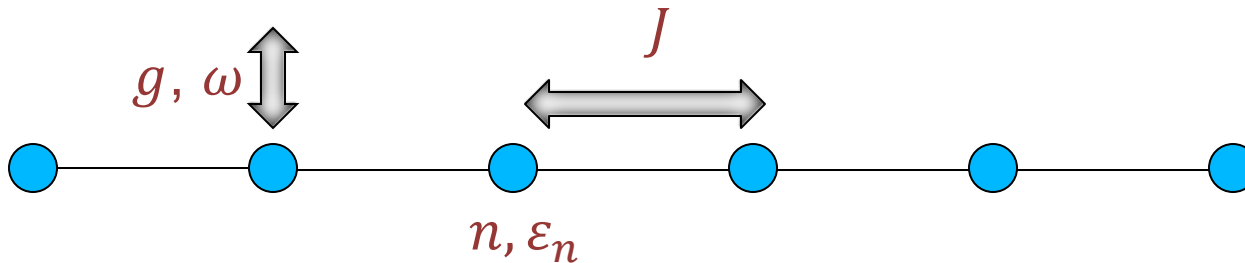


Science Fund
of the Republic of Serbia

PNAS 119(3):e2113967119 (2022)

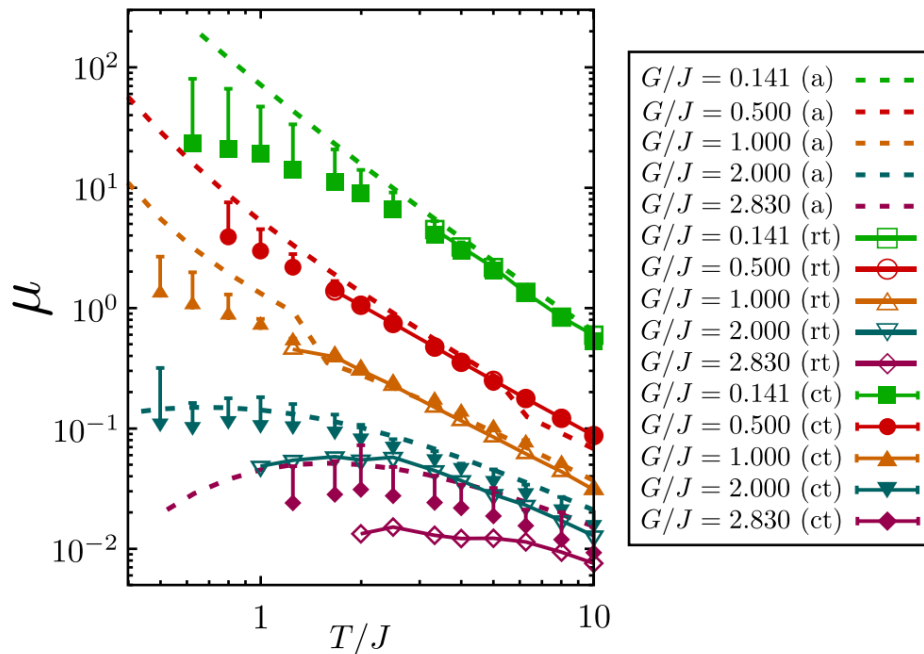
Numerically exact approaches

- ❖ Direct evaluation of current-current correlation function starting from the Hamiltonian of the system.
- ❖ Example: Holstein model
- ❖ Direct diagonalization - feasible up to only ~ 3 sites
- ❖ Other approaches:
 - Path-integral Quantum Monte Carlo
 - Hierarchical equations of motion
 - Time dependent density matrix renormalization group
 - ...



Quantum Monte Carlo for mobility of Holstein polaron

- ❖ Evaluation of integrals involving Gaussian functions using Monte Carlo sampling
- ❖ Essentially single processor code
- ❖ Straightforward parallelization: to obtain Monte Carlo statistics



S. Miladić, N. Vukmirović,
Phys. Rev. B 107, 184315 (2023).

Summary

- ❖ Different physics requires different algorithms even for the same problem of charge transport in semiconducting material.
- ❖ HPC is indispensable in each case.
- ❖ New physical insights:
 - HPC
 - physical intuition
 - algorithm and code development

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